

Voltage Drop Calculations

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Abstract—This document serves a guide for the selection of conductor sizes taking into account the effect of voltage drop. Several methods of determining the voltage drop will be presented including a method for determining the voltage drop where unbalanced load current conditions occur.

Index Terms—voltage drop, unbalanced load

I. INTRODUCTION

In 2008, I have written some articles on voltage drop calculations as a tool in the selection of the correct cable sizes.

- 1) Voltage Drop Calculation (Pangonilo, 2008) [1];
- 2) Voltage Drop Calculation Part 2 (Pangonilo, 2008) [2];
- 3) Voltage Drop Calculation Part 3 (Pangonilo, 2008) [3];

This tutorial is an update of above-mentioned tutorials combined into single article complete with a downloadable pdf. The calculations will be based on IEC standards or its derivatives. However, the calculations will likewise be applicable to North American standards using the correct cable parameters.

II. OBJECTIVES

The selection of cables depends on several factors

- 1) Current carrying capacity of conductor with derating factors applied;
- 2) Conductor operating temperature as an effect of the load current;
- 3) Voltage drop during normal operation is within acceptable limits;
- 4) Voltage dip during motor starting, if applicable, is within acceptable limits;
- 5) Short-circuit withstand to ensure that conductor will not melt during a short-circuit condition.

Only item 4—voltage drop will be considered in this tutorial.

At the end of this tutorial, the electrical engineer will be able to select the correct cable size taking into account the effect of voltage drop.

III. BASIS OF CALCULATION

Cable selection is an important part of electrical installation design. Selecting the correct size of cable could lower initial cost, lower operating cost, better voltage regulation notwithstanding the safety factor.

Cable parameters are provided in values per unit length, usually ohms per kilometer(Ω/km) for IEC or miles(Ω/mi) for North American Standards.

The following table illustrate the typical parameters for industrial cables

Table I: 600/1000 V. Cu/EPR/CSP/GSWB run on open trays or enclosed in air. 1 to 6 x single, 3 or 4 cores

Nominal conductor area (mm^2)	Single cores in trefoil		3 cores		Approx. armouring resistance
	Resistance at 90°C (Ω/km)	Reactance at 50Hz	resistance at 90°C (Ω/km)	Reactance at 50Hz	GSWB at 60°C 3-4 cores (Ω/km)
1.5	15.6	0.185	15.6	0.118	46.2
2.5	9.64	0.173	9.64	0.111	51.3
4	5.99	0.163	5.99	0.108	60.3
6	3.97	0.153	3.97	0.105	30.5
10	2.35	0.148	2.35	0.0983	36.7
16	1.48	0.134	1.48	0.0933	23.1
25	0.936	0.125	0.936	0.0892	28.1
35	0.674	0.121	0.674	0.0867	10.43
50	0.499	0.118	0.499	0.0858	11.81
70	0.344	0.112	0.344	0.0850	13.61
95	0.271	0.108	0.271	0.0825	10.87
120	0.214	0.106	0.214	0.0808	11.92
150	0.175	0.105	0.175	0.0808	7.38
185	0.140	0.105	0.140	0.0808	8.15
240	0.108	0.103	0.108	0.0800	8.94
300	0.087	0.101	0.087	0.0800	10.10
400	0.069	0.0992	0.069	0.0795	10.00

IV. DERIVATION OF VOLTAGE DROP FORMULA

$$V_d = IR\cos\Phi + IX\sin\Phi \quad (1)$$

where:

V_d = Voltage drop per phase, volts

Φ = Load power factor angle

I = Current, A

R = Cable resistance, Ω

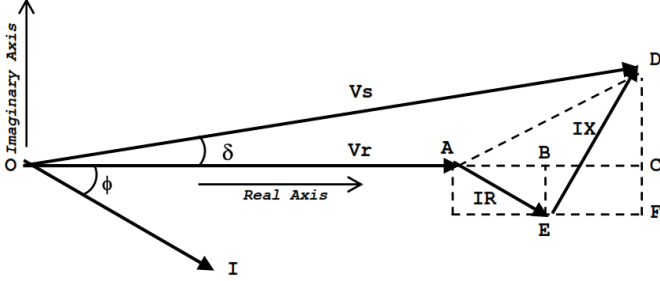
X = Cable reactance, Ω

$IR\cos\Phi$ = Voltage drop component on the cable resistance

$IX\sin\Phi$ = Voltage drop component on the cable reactance

This formula is an approximation. We shall derive this formula to understand it better. Consider the following vector diagram in Figure 1.

Figure 1: Voltage drop vector diagram



where:

- $V_s = OD =$ Sending end voltage per phase
- $V_r = OA =$ Receiving end voltage per phase
- $I =$ Load current
- $R =$ Cable resistance
- $X =$ Cable reactance
- $\Phi =$ Load power factor angle
- $AB = IR\cos\Phi$
- $BE = CF = IR\cos\Phi$
- $BC = EF = IX\sin\Phi$
- $DF = IX\cos\Phi$

Let us analyze Figure 1 along its horizontal axis,

$$AC = AB + BC = AB + EF \quad (2)$$

From eq. 2 and Figure 1, it can be determined that

$$AB = IR\cos\Phi \quad (3)$$

Likewise,

$$BC = IX\sin\Phi \quad (4)$$

Substituting eq. 3 and 4 in eq. 2

$$AC = IR\cos\Phi + IX\sin\Phi \quad (5)$$

The next step of the derivation is to analyze Figure 1 along its vertical axis,

$$DC = DF - CF = DF - BE \quad (6)$$

From eq. 6 and Figure 1, it can be determined that

$$DF = IX\cos\Phi \quad (7)$$

Likewise,

$$CF = IX\sin\Phi \quad (8)$$

Substituting eq. 7 and 8 in eq. 6

$$DC = IX\cos\Phi - IR\sin\Phi \quad (9)$$

Taking another look at Figure 1, the sending end voltage (V_s) is equal to

$$V_s = OD = \sqrt{(OA + AB + BC)^2 + (DF - BE)^2} \quad (10)$$

Unless the cable is very long, the imaginary axis component of the voltage is very small compared to the real axis component.

$$(OA + AB + BC) \gg (DF - BE) \quad (11)$$

thus the sending end voltage will be

$$V_s \approx (OA + AB + BC) \quad (12)$$

This was the reason why I mentioned earlier that the voltage drop formula in eq. 1 is an approximation.

From Figure 1, eq. 3 and eq. 8, the sending end voltage could be calculated as

$$V_s = V_r + IR\cos\Phi + IX\sin\Phi \quad (13)$$

volts/phase

From the eq. 13, the voltage drop on a per phase basis will be

$$V_d = V_s - V_r \quad (14)$$

Thus the voltage drop equation as mention in eq. 1 is

$$V_d = IR\cos\Phi + IX\sin\Phi \quad (15)$$

volts/phase per unit length

That is how the voltage drop formula was derived. Normally, voltage drop is expressed as a percentage of the sending end line-to-line voltage. The formula will be

$$\%V_d = \frac{\sqrt{3}I(R\cos\Phi + X\sin\Phi)}{V} \% \quad (16)$$

where:

$V =$ sending end line-to-line voltage

$R = rl$, where r is the unit resistance in Ω/km

$X = xl$, where x is the unit reactance in Ω/km

The values of R and X shall be as published in cable data publications.

V. SAMPLE CALCULATIONS

After understanding how the voltage drop formula was derived in Section IV. We are now ready to get started with some real world examples to be able to appreciate voltage drop calculation better as it may have already been a part of what you have already done or currently doing in your design. We shall be presenting an IEC standard equipment in our sample calculation.

A 120 mm² 3core XLPE insulated cable 120m in length supplies a 110 kW induction motor that has a starting current of 6 times the full-load current of 180 amps. The starting power factor is 0.25 lagging. If the sending end line-to-line voltage is 400 volts, the specific resistance r and reactance x for the cable are 0.214 and 0.0808 Ω/km respectively at 90°C and 50 Hz. Find the percentage volt-drop on starting the motor.

Solution:

From Table I, a 120 m × 120 mm² cable, its parameters are:

$$R = rl = 0.214 \times 120/1000 = 0.02568 \Omega/\text{phase}$$

$$X = xl = 0.0808 \times 120/1000 = 0.009696 \Omega/\text{phase}$$

For the 110kW motor, the starting current is:

$$I = 6 \times 180.0 = 1080.0 \text{ A}$$

The power factor is:

$$\cos\Phi = 0.2500$$

therefore

$$\sin\Phi = 0.9682$$

Let us assume the sending voltage remains constant at 400 volts,

Substituting the above values in eq. 16

$$\%V_d = \sqrt{3} \left(\frac{1080}{400} \right) (0.02568 \times 0.25 + 0.009696 \times 0.9682) \times 100\% \quad (17)$$

$$\%V_d = 4.6765 \times (0.00642 + 0.006738) \times 100\% \quad (18)$$

$$\%V_d = 7.39\% \quad (19)$$

From eq. 14, we could calculate the value of the receiving end voltage

$$V_r = V_s - V_d \quad (20)$$

Therefore, the receiving end voltage can be calculated as

$$V_r = \frac{400}{\sqrt{3}} \times \left(1 - \frac{7.39}{100} \right) \quad (21)$$

$$V_r = 213.867 \text{ volts/phase} \quad (22)$$

Another method of calculating the receiving end voltage is to use Figure 1

$$AB = 1080.0 \times 0.02568 \times 0.2500 = 6.9336 \text{ volts/phase}$$

$$BE = 1080.0 \times 0.02568 \times 0.9682 = 26.8537 \text{ volts/phase}$$

$$EF = 1080.0 \times 0.009696 \times 0.9682 = 10.1392 \text{ volts/phase}$$

$$DF = 1080.0 \times 0.009696 \times 0.2500 = 2.6179 \text{ volts/phase}$$

To calculate V_r using the values above

$$V_r = V_s - (AB + EF) \quad (23)$$

$$V_r = \frac{400}{\sqrt{3}} - (6.9336 + 10.1392) \quad (24)$$

$$V_r = 213.867 \text{ volts/phase} \quad (25)$$

It checks out that the result of eq. 22 and eq. 25 will be the same.

The typical minimum starting voltage for motors is 80% × rated voltage. The result of the voltage drop calculation from above

example is less than 20%, therefore the motor will accelerate to full speed easily.

In practice, since the actual motor parameters are not known during the design stage, the minimum starting voltage is raised to 85% to provide a factor for safety.

VI. CABLE SELECTION

Our example in Section V, we have only considered a single cable and checked only the voltage drop at starting. In actual practice, we need to compare several cable sizes then select the cable that provides optimum design consideration.

In this example, we shall select several cables from 35 mm² up to 120 mm². Likewise, we shall consider, not only the starting condition but the running condition of the motor as well. You might be asking, why we selected a motor in our example. The obvious reason is, a motor is a dynamic load. A motor circuit load varies from starting up to the normal running condition, thus there will be multiple considerations when selecting the cable for a motor circuit.

Table II: BS5467 SWA/PVC Cable IEC 60502 600/1000V Current Ratings and Electrical Data (3-core) [6]

Size, mm ²	Current ratings in Air, A
35	172
50	209
70	263
95	324
120	376

The first step in the cable selection process is to select the cable with a current carrying capacity that is equal to or greater than the operating full load current of the motor which is 180A.

From Table II, it is very clear that a 35mm² has a current carrying capacity lower than the full load current of the motor thus not suitable for the application. However, we will include it in our analysis anyway.

The criteria given below shall be our bases in the cable selection according to voltage drop:

$$\text{Voltage drop during starting} \leq 15\%$$

$$\text{Voltage drop during running} \leq 5\%$$

Using the same values from our previous example in Section V:

$$\text{Voltage sending end} = 400 \text{ volts}$$

Motor Running:

$$I_{\text{running}} = 180 \text{ A}$$

$$PF_{\text{running}} = \cos\Phi_{\text{running}} = 0.87$$

$$\sin\Phi_{\text{running}} = 0.493$$

$$\text{Motor Starting } I_{\text{starting}} = 6 * 180 = 1080 \text{ A}$$

$$PF_{\text{starting}} = \cos\Phi_{\text{starting}} = 0.25$$

$$\sin\Phi_{\text{starting}} = 0.968$$

Using the formula in eq. 16, tabulate the results similar to Table III.

Table III: Cable Selection

Size	r ₅₀	x ₅₀	R	X	%V _{dr}	%V _{ds}
35	0.674	0.0867	0.0809	0.0104	5.88	14.17
50	0.499	0.0858	0.0599	0.0103	4.46	11.66
70	0.344	0.0850	0.0413	0.0102	3.19	9.44
95	0.271	0.0825	0.0325	0.0099	2.59	8.28
120	0.214	0.0808	0.0257	0.0097	2.11	7.39

where:

%V_{dr} = Voltage drop with motor running

%V_{ds} = Voltage drop during motor starting

In Table III. the 35 mm² cable passes the motor starting voltage drop but fails the motor running voltage drop which makes this cable to be eliminated from our selection list. Also, as previously mentioned, the current carrying capacity of the cable is lower than the operating full load current of the motor.

The 70 mm², 90 mm² and 120 mm² cables satisfy all our design conditions, however, selecting any of these cables will make the installation more expensive than required. The 50 mm² cable satisfies all conditions and cost of the project.

Cable selection considerations does not stop here, there are more that need to be considered such as operating temperature and short-circuit withstand which is not within the scope of this tutorial.

VII. SIMPLIFIED CALCULATIONS

This calculation of voltage drop is very straight forward. Most cable manufacturers provide the unit voltage drop in millivots per ampere-meter (mV/A.m). To calculate the total voltage drop in a circuit length, use eq. 27.

$$V_c = \frac{1000 \times V_d}{L \times I} \quad (26)$$

$$V_d = \frac{L \times I \times V_c}{1000} \quad (27)$$

$$V_p \geq \text{sum of } V_d \text{ on circuit run}$$

where:

V_c is the the millivolt drop per ampere-meter route length of circuit, as provided by cable manufacturers, in millivolts per ampere meter (mV/A.m);

V_d = actual voltage drop, in volts

V_p = permissible voltage drop on the circuit run, e.g. 5% of supply voltage, in volts

L = route length of circuit, in meters

I = the current to be carried by the cable, in amperes.

To provide an example, let us assume a 50 meter long, 10mm² mutlicore conductor at 90°C operating temperature is 4.05 mV/A.m supplying aload of 25A. Calculate the voltage drop.

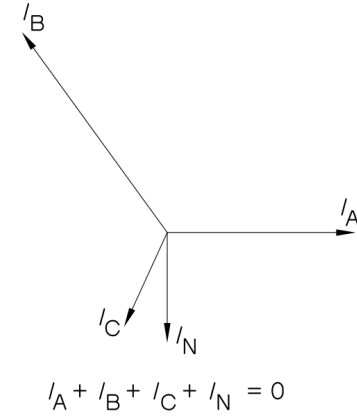
Substituing the above values in eq. 27

$$V_d = \frac{50 \times 25 \times 4.01}{1000} = 5 \text{ V} \quad (28)$$

VIII. UNBALANCED THREE PHASE CIRCUIT

For unbalanced three (3) phase circuits, current will be flowing through the neutral conductor, refer to Figure 2.

Figure 2: Unbalanced Three Phase Circuit Phasor Diagram



A conservative solution to the voltage drop calculation on unbalanced circuit would be to assume a balanced three-phase load. Calculate the voltage drop using the current flowing in the heaviest-loaded phase. In most cases this is applicable if the unbalance conditions are inconsistent or intermittent.

However, where the currents in each phase can be shown to be of different magnitudes for consistent periods, the voltage drop calculations can be performed on a single-phase basis by geometrically summing the voltage drop in the heaviest loaded phase and the voltage drop in the neutral, as follows:

$$V_d = \text{voltage drop in heaviest loaded phase} + \text{voltage drop in neutral}$$

$$V_d = I_A \times L_A \times Z_{CA} + I_N \times L_N \times Z_{CN} \quad (29)$$

The voltage drop in each phase conductor can then be assessed considering all parameters such as conductor material, size, temperature and length, the magnitude and phase angle of the current flowing in each conductor, and the phase angle of the load.

IX. SINGLE PHASE CIRCUITS

In the previous sections, the voltage drop calculations presented were intended for three (3) phase circuits.

To calculate the voltage drop of single phase circuits, there will be a slight change in eq. 16.

$$\%V_d = \frac{2 \times I \times (R \cos \Phi + X \sin \Phi)}{V} \% \quad (30)$$

where:

V = sending end line-to-line voltage

R = rl, where r is the unit resistance in Ω/km

X = xl, where x is the unit reactance in Ω/km

For direct current applications, in eq. 31 the reactive component of the voltage drop will be deleted as the power factor is always unity.

$$\%V_d = \frac{2 \times I \times R}{V} \% \quad (31)$$

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