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# Influence of unbalanced voltage supply on efficiency of three phase squirrel cage induction motor and economic analysis

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#### Abstract

The negative effect of unbalanced voltage upon the losses and efficiency of a three phase squirrel cage induction motor is investigated. The financial losses caused by unbalanced voltage of the power supply are determined using an analytical–statistical method. The analytical method is verified by experimental test.

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Keywords: Induction motors; Unbalanced voltage; Efficiency

## 1. Introduction

The performance of a three phase induction motor supplied by unbalanced voltage is far from desirable and can damage the motor. The negative sequence component of the voltage due to voltage unbalance generates a reverse rotating air gap field in the opposite direction of rotor rotation. All undesirable effects of the unbalanced voltage upon the performance of the motor are produced by this reverse field.

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## Nomenclature

 $\begin{array}{ll} V_{\mathrm{Ta}}, V_{\mathrm{Tb}}, V_{\mathrm{Tc}} \mbox{ equivalent terminal voltage of phase a, b, c} \\ I_{\mathrm{Ta}}, I_{\mathrm{Tb}}, I_{\mathrm{Tc}} \mbox{ equivalent current of phase a, b, c} \\ R_{\mathrm{r}}, R_{\mathrm{s}} \mbox{ rotor bars and stator winding resistance} \\ X_{\mathrm{r}}, X_{\mathrm{s}} \mbox{ rotor bars and stator winding reactances} \\ V_{\mathrm{Sa}}, V_{\mathrm{Sb}}, V_{\mathrm{Sc}} \mbox{ equivalent stator winding voltage of phase a, b, c} \\ I_{\mathrm{Sa}}, I_{\mathrm{Sb}}, I_{\mathrm{Sc}} \mbox{ equivalent stator winding current of phase a, b, c} \\ R_{\mathrm{c}} \mbox{ core losses equivalent resistance} \\ X_{\mathrm{m}} \mbox{ magnetization reactance} \\ P_{\mathrm{FW}} \mbox{ rotational losses} \\ P_{\mathrm{sh}} \mbox{ shaft power} \end{array}$ 

The reduction in efficiency of a three phase induction motor supplied by unbalanced voltages was studied in Ref. [1]. This leads to a considerable temperature rise and shorter life time of the machine [2]. Supplying the motor with unbalanced voltages also decreases the rating of the motor [3]. A simple and brief method was applied in order to study the impact of unbalanced voltages on the losses and its negative effect on the insulation of the motor [4]. Protection of the motor against these risks and adjusting the relays were proposed in Ref. [5]. In spite of this, the following points are notable:

- 1. The impact of unbalanced voltage on the performance has been normally described qualitatively, not quantitatively or numerically or using convenient curves.
- 2. The impact of the unbalanced voltage upon the load characteristics and consumed power has not received enough attention.
- 3. The adequacy of the curves and definitions in the standards has not been checked precisely.

For the above mentioned reasons, the subject has been revisited in recent years. In Refs. [6,7], the impact of unbalanced voltage on the losses of the motor has been investigated. In Ref. [6], the effect of 0-5% unbalanced factors (based on the NEMA definition) has been discussed. For the same magnitude of the unbalanced voltage, the method of producing the unbalance of the terminal voltages has a considerable effect on the stator and rotor copper losses. An increase in the unbalanced voltage at the terminals can increase rotor losses more than stator losses. The reason is that the rotor currents have a larger deviation than the stator currents. In addition, the resistance of the rotor bars is higher for unbalanced voltage [8]. For an unbalanced factor higher than 5%, the increase of the losses is considerable, and this leads to a higher temperature rise that could damage the motor. Therefore, use of a motor connected to a large unbalanced voltage is not normally allowed.

In Ref. [9], the impact of unbalanced voltage on the efficiency has been studied experimentally, and the efficiency curves versus load with balanced voltages; with unbalanced winding and 2.5% unbalanced voltage have been given. The efficiency of the motor at this unbalanced voltage is about 2% (at all loads) lower than that of the balanced voltage case. In Ref. [10], the impact of

the unbalanced voltage on the efficiency and power factor has been given. It indicates that for any unbalanced voltage factor at a given voltage, there are different cases for the terminal voltages, and each case has particular results. By increasing the amplitude of the positive sequence component of the voltage, a reduction of the power factor has been observed. In the same paper, a simple economic study has also been performed, and the overload in the power system and extra cost for consumers with 4% unbalanced voltage has been investigated. In Ref. [10], the temperature rise due to unbalanced voltages has been studied experimentally.

In Ref. [11], a combination of the finite element (FE) technique and symmetrical components analysis has been used to investigate the temperature rise and life time of the insulation. The hot spots for different loads have also been obtained. In Ref. [6], the derating factors of the motor using positive and negative sequence equivalent circuits and symmetrical component analysis have been determined. It has been shown that knowledge only of the percentage of the unbalanced voltage cannot express all existing conditions on the motor, and the type of unbalance must also be specified. In Ref. [12], the total losses of the machine have been experimentally measured, and derating factors for two cases have been introduced. In the first case, the maximum losses were assumed equal to the rated losses, and in the second case, the maximum current of the different phases was assumed equal to the rated current. In Ref. [13], one of the most complete investigations in the derating of the motor has been conducted out using a complex unbalanced voltage factor. For a given motor, two derating curves have been introduced, one for the worse and the other for the best case, and the region between these two curves consists of all derating factors.

The test results of a motor have been reported in Ref. [14], and the corresponding unbalanced currents have been drawn. The following cases have been studied in the above mentioned paper: (1) a relatively low unbalanced voltage factor for low loads, (2) current unbalance for 2.5% unbalanced factor (about 30%), (3) negligible unbalance due to unsymmetrical windings of the machine and (4) a small inherent unbalanced current. In Ref. [15], the effects of currents from the unbalanced voltage have been discussed. It has been shown that during low loads (where the speed of the motor is near the synchronous speed), the unbalanced current rises to 10 times that of the unbalanced voltage factor. This difference is reduced by raising the slip(s).

An unbalanced voltage supply leads to a reduction in the torque of the induction motor. An increase in the motor's current forces the user to derate the motor. Other undesirable effects in the induction motor resulting from the unbalanced voltage supply are studied in this paper. This paper extends the effects of unbalanced voltages upon the losses of the three phase squirrel cage induction motor to an economic analysis.

#### 2. Losses in electrical motors

The equivalent stator and rotor currents and corresponding copper losses at any load or slip values are determined by proper equivalent circuit calculations. The friction and windage losses are usually 1-2% of the rated output. The no load core losses are markedly dependent on many features of design, but a rough average figure is 2-3% of the rated output. In general, the percentage core losses decreases if the slot openings or the ratio of rotor to stator slots is made smaller or if the air gap length in increased. A large proportion of the core loss and the stray load losses is

normally due to slot frequency pulsation losses, so that high quality silicon steel is usually employed for core laminations. Typical values of stray core loss are 1-2% of the output [16,17].

## 3. Effect of unbalanced voltage on losses

In addition to the positive sequence component, the negative sequence component of the current and the difference of the phase currents in the unbalanced voltage case is one of the major reasons for the increase in losses in the machine. If the total losses of an induction machine are divided into three parts, mainly the copper losses (stator and rotor), core losses and mechanical losses, the unbalanced voltage has the largest influence on the copper losses and the lowest influence on the mechanical losses.

In order to analyze the performance of a three phase motor supplied by an unbalanced voltage and visualize its effect upon the losses of the machine, a 7.5 kW three phase induction motor with specifications as given in Table 1 is proposed. The sequence per phase equivalent circuit of a three phase induction motor has been shown in Fig. 1.

When the motor operates with 4% slip and rated output power, the performance conditions are summarized in Table 2.

In the loss estimation for the unbalanced voltage supply, stator copper losses, rotor ohmic losses, core losses and mechanical losses (friction and windage) have been taken into account and stray losses have been ignored. Mechanical losses of 100 W have been reported by the manufacturer. Since the core losses resistance is also known, its variations in both balanced and unbalanced conditions may be taken into account.

The proposed motor is analyzed over full load and 0–6% unbalanced voltage (based on the complex voltage unbalance factor, CVUF, definition), and the different losses are evaluated. Of course, with the definition of CVUF, in addition to the unbalance angle, determination of the average line voltage is necessary in order to obtain precise results from the analysis. Here, the unbalance of the terminal voltage will be considered with an angle  $\theta = 120^{\circ}$  and average voltage equal to 380 V. As usual, it is assumed that the average terminals voltage in industrial plants is lower than the rated value.

Specifications of a three phase induction motor									
Voltage (V)	$R_{\rm s} \left( \Omega \right)$	$R_{\rm r}$ ( $\Omega$ )	$X_{\rm s} \left( \Omega \right)$	$X_{\rm r}$ ( $\Omega$ )	$R_{\rm c} \left( \Omega \right)$	$X_{\mathrm{m}}\left(\Omega\right)$	No. of poles		
380	2.62	1.80	2.90	4.35	1579.4	93.368	4		
	2.625	1.80	2.903	4.355	1579.4	93.368	4		





Fig. 1. Sequence equivalent circuit.

Stator current (phase a), A	Stator line current (phase a), A	Stator copper losses, W	Rotor ohmic losses, W	Core losses, W	Mechanical losses, W	Total losses, W	
9	15.6	640.3	317	227.8	100	1285	

There is a suitable method [6] that analyzes the motor in the phase frame. First, the terminal voltages and current of the motor are determined, and then these values are used to compute the stator and rotor losses and the converted shaft power.

The power converted to shaft power  $(P_{conv})$  is given by

$$P_{\rm conv} = V_{\rm Ta} \cdot (I_{\rm Ta})^* + V_{\rm Tb} \cdot (I_{\rm Tb})^* + V_{\rm Tc} \cdot (I_{\rm Tc})^*$$
(1)

If the rotational losses ( $P_{\rm FW}$ ) are known, then the shaft power ( $P_{\rm sh}$ ) can be determined as follows:

$$P_{\rm sh} = P_{\rm conv} - P_{\rm FW} \tag{2}$$

The rotor copper loss  $(P_r)$  and stator copper losses  $(P_s)$  are as follows:

$$P_{\rm r} = |I_{\rm Ta}|^2 \cdot R_{\rm r} + |I_{\rm Tb}|^2 \cdot R_{\rm r} + |I_{\rm Tc}|^2 \cdot R_{\rm r}$$
(3)

$$P_{\rm s} = |I_{\rm sa}|^2 \cdot R_{\rm s} + |I_{\rm sb}|^2 \cdot R_{\rm s} + |I_{\rm sc}|^2 \cdot R_{\rm s}$$
(4)

The total input power is

Table 2

....

$$P_{\rm in} = \operatorname{Re}[V_{\rm sa} \cdot (I_{\rm sa})^* + V_{\rm sb} \cdot (I_{\rm sb})^* + V_{\rm sc} \cdot (I_{\rm sc})^*]$$
(5)

With the above mentioned conditions and the use of phase domain analysis, the stator and rotor copper losses of the proposed motor for 0-6% unbalanced voltages are obtained as shown in Figs. 2 and 3. It is realized that the variations of the stator and rotor losses versus unbalanced voltage are not linear. It must be noted that the losses in the unbalanced condition have been normalized with the corresponding losses for the balanced condition, and the shapes of the curves depend on the square of the current difference with the balanced condition. Fig. 2 shows the stator copper losses of the different phases of the machine. As shown, the losses of phase a and b are increased by increasing the voltage unbalance, while the losses of phase c decrease. On the other hand, the rate of increase of the losses of phase b is larger than that for phase a. For a 5% unbalanced voltage factor, the copper losses of phase a are 12%, phase b are 160% and phase c are 68%. The reason for these differences in losses comes from the unbalanced currents. Current unbalance and deviation of the stator currents from each other lead to the copper losses variations. For any voltage unbalance, the motor operates at full load and different slip. This arises from very large differences in the negative and positive sequence equivalent impedances leading to an excessive temperature rise of the machine. The copper losses of the phases in the rotor of the proposed machine for voltage unbalances of 0-6% are given in Fig. 3. The copper losses of phase a, b and c follow the same trend as those in the stator. However, although the losses of the rotor for phases a and b are approximately the same, this is not true in the stator.

The total losses at 5% voltage unbalance are about 12% larger than that of the balanced case. If the motor operates a long time under such conditions, this can produce serious damage, and thus,

it becomes necessary to derate the motor. The temperature rise is different for the windings of the different phases and is not uniform within the machine. This can result in premature damage to the machine.

As shown in Fig. 3, increasing the voltage unbalance at the motor terminals leads to an increase of the rotor losses compared to the stator losses. The reason is that the rotor equivalent circuit is more sensitive to the slip.

Figs. 2–4 present the losses variations with different voltage unbalances and under full load conditions. In order to study the increase of losses under other loads, loads of 50–125% of full load operating with 3–6% voltage unbalance are analyzed. Under these conditions, the operating slip is obtained and the total losses are then calculated. The results are presented in Fig. 5. The increase of losses at each load is determined in comparison with the balanced voltages case with the same load. Fig. 5 indicates that regardless of the load, the total losses of the machine with 6% voltage unbalance are larger than those of the 3% case. Also, the influence of the voltage unbalance on the increase of the losses is considerably lower at low loads. For 50% full load, the losses at 3% voltage unbalance are about 1.5% and at 6% voltage unbalance it is about 12%, while these values for the fully loaded motor are 8.5% and 14.5% respectively. The reason is that for low loads, the stator and rotor conductor losses are also low, but the core and mechanical losses remain approximately fixed. Thus, the ratio of the loss increase with unbalanced voltages will be smaller. For larger loads, the conductor losses are also large, and this ratio is increased.

In addition to the temperature rise, the voltage unbalance reduces the efficiency of the motor. The excess heat can be dissipated using different methods, allowing safe operation of the machine; however, the efficiency reduction cannot be compensated. This efficiency reduction is undesirable, and it is particularly unacceptable for energy efficient motors. Here, a three phase induction motor is analyzed under voltage unbalance, and precise numerical values of efficiency are given. Fig. 6 presents the efficiency of the 7.5 kW motor under 0-6% voltage unbalance with phase angle equal



Fig. 2. Copper losses of different stator phases with increasing unbalanced voltage factor.



Fig. 3. Copper losses of rotor for different phases with increasing voltage unbalance factor.



Fig. 4. Variations of total losses for different loads.

to 120° (defined CVUF) and average terminals voltage equal to 360 V. As seen, an increase in the terminal voltage unbalance reduces the efficiency of the motor. For instance, for a balanced case, the efficiency is 85.4%, while at 5% voltage unbalance, it is 83.9%.

At any operating condition, the above mentioned method may be employed, and the operating slip evaluated for the rated load and the given terminal voltages. The losses and efficiency of the



Fig. 5. Total losses of the machine at 6% and 3% voltage unbalance.



Fig. 6. Variations of efficiency of motor under voltage unbalance.

motor are then calculated. It is clear that no limitation has been considered for the currents, and in particular, the currents may exceed the rated values, which may damage the motor. Fig. 7 shows the line current of the motor for the above mentioned conditions. According to Fig. 6, for the case with voltage unbalance, at least one of the currents is larger than the rated current. Since a current higher than the rated current is not permissible, the upper limit of the currents in the unbalanced case is the rated current with balanced voltages. Fig. 5 has been obtained with no derating factors. In order to protect the machine against damage, it must be derated in addition to



Fig. 7. Variations of motor current under different voltage unbalance conditions.



Fig. 8. Variations of efficiency of motor for different unbalances and applying required derating factor.

producing a reduction in the efficiency that must be calculated. Based on this assumption, the efficiency of the proposed 7.5 kW machine for voltage unbalance factors of 0-6% with a complex unbalanced factor and average voltage equal to 360 V are shown in Fig. 7. The following are noticeable in Fig. 8:

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- 1. At any voltage unbalance, the efficiency of the motor is higher than that in Fig. 6. The reason is that the machine has been derated, and the currents are in the permissible range. Therefore, the losses of the machine are limited, and the efficiency is higher. For instance, at 3% voltage unbalance, the efficiency of the machine is equal to the efficiency of the balanced case (85.4%).
- 2. In spite of an increase of the voltage unbalance, the efficiency does not necessarily decrease. Referring to Fig. 8, for up to 3% voltage unbalance, the efficiency rises, but thereafter, the efficiency decreases for increasing voltage unbalance. The reason is that the motor has been derated for the voltage unbalance case, which protects the windings and increases the efficiency of the machine.

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No.	$V_{ab}$	$V_{\rm bc}$	$V_{\rm ca}$	Unb.	Ia	Ia	Ib	Ib	Ic	Ic	$P_{\rm loss}$	Eff.
				(%)	(Anal.)	(Exp.)	(Anal.)	(Exp.)	(Anal.)	(Exp.)	(%)	(%)
1	372	378	368	1.43	15.18	15	17.13	16.7	15.33	15	102.4514	85.05
2	370	382	361	2.97	14.71	14.2	18.63	18.2	14.87	14.6	105.1611	84.74
3	373	379	364	2.15	15.31	15.1	17.77	17.5	14.74	14.5	103.3556	84.94
4	376	375	369	1.16	16.03	15.7	16.56	16.5	14.92	14.7	101.9704	85.11
5	377	374	370	1	16.28	16.1	16.32	16	14.95	14.5	102.0973	85.12
6	369	372	370	0.45	15.57	15.3	16.22	16	16.06	16	102.8171	85.02
7	372	378	365	1.8	15.28	15.1	17.54	17.4	15	14.8	103.1728	84.97
8	371	384	363	3.04	14.5	14.1	18.57	18.3	14.94	14.7	104.586	84.8
9	373	383	368	2.22	14.67	14.5	17.72	17.4	15.18	15	102.6996	85.03
10	375	383	371	1.68	14.81	14.5	17.26	17.2	15.21	15.1	101.6249	85.16
11	375	371	366	1.26	16.54	16.2	16.54	16.3	14.79	14.6	103.0739	84.98
12	378	371	363	2.07	17.01	16.7	16.93	16.8	14.03	13.5	103.8296	84.89
13	377	368	366	1.8	17.18	17	16.18	16	14.59	14.1	103.4949	84.93
14	374	370	367	1	16.52	16.2	16.3	16.2	15.07	15.1	103.0272	84.99
15	376	368	370	1.26	16.94	16.6	15.63	15.4	15.21	15.1	102.772	85.02
16	374	370	367	1	16.52	16.1	16.3	16	15.07	14.8	103.0272	84.99
17	376	365	364	2.08	17.51	17	16.14	16	14.57	14.2	104.4226	84.82
18	379	365	360	2.99	18.02	17.7	16.7	16.3	13.69	13.5	105.6599	84.66
19	381	370	364	2.5	17.5	17.2	16.69	16.4	13.79	13.5	104.1479	84.86
20	382	375	370	1.68	16.75	16.5	16.37	16.1	14.24	14	101.8545	85.13
21	377	372	369	1.16	16.55	16.5	16.21	16	14.85	14.7	102.2615	85.08
22	379	362	363	3	18.31	18	15.95	15.7	14.15	14	105.5393	84.67
23	380	367	364	2.61	17.74	17.4	16.34	16.3	13.98	13.8	104.3051	84.83
24	381	373	366	2.05	17.03	16.7	16.73	16.5	13.95	13.7	103.1486	84.98
25	380	377	370	1.5	16.25	16	16.62	16.4	14.47	14.1	101.6397	85.16
26	366	377	366	1.98	14.65	14.3	17.44	18.1	15.96	15.8	103.863	84.89
27	367	381	362	2.97	14.21	14.2	18.31	18	15.62	15.5	104.7984	84.78
28	369	381	366	2.42	14.52	14.4	17.86	17.5	15.54	15.3	103.7914	84.91
29	371	381	368	2.05	14.69	14.2	17.54	17.4	15.47	15.4	102.9447	85.01
30	373	382	375	1.42	14.59	14.2	16.66	16.5	15.95	15.5	101.1813	85.21

 Table 3

 Performance of the proposed three phase induction motor under the measured unbalanced voltage of the power system

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As an overall conclusion, it is clear that increasing the voltage unbalance reduces the efficiency of the motor. If a motor operating under voltage unbalance is derated correctly, the efficiency may be increased.

#### 4. Experimental study of unbalanced voltage

The influence of unbalanced voltage on the performance of a three phase induction motor with specifications summarized in Table 1 have been experimentally investigated in order to verify the obtained results.

The unbalanced voltage at the terminal of the motor has been measured by a digital voltmeter over a 6 day period and 5 times a day from 10 AM to 10 PM as shown in Table 3. Also, the line currents of the motor have been measured by a hook type ammeter.

The results obtained by the analytical method have also been determined experimentally (Table 3). It shows that the deviation of the line currents is visible in all unbalance values. This difference is higher for larger unbalance. However, in all cases, the phase current is larger than the rated current (15.6 A), i.e. the motor is always overloaded. The losses in the unbalanced condition have been compared with those in the balanced condition, and it has been found that the losses increase by 2-6% over the proposed period.

The maximum unbalanced voltage is 3%, the values of which have been summarized in Table 4. The analysis results show that except for the amplitude of the current, the other performance indices are similar. In other words, the losses of the motor and also the efficiency are almost the same in both cases. It is noted that the reason for such equality in performance comes from the equality of the average value of the terminal voltages in addition to the unbalanced voltage.

In the two above mentioned cases, the amplitudes of the line currents are different, and the currents strongly depend on the voltage amplitudes. A slight change of the voltages will cause rather large changes of the currents. For equal unbalanced voltage and average voltages, the currents of the machine can still differ. This is visible from the analysis and the experimental results.

#### 5. Economic analysis

From the consumer's point of view, efficiency reduction means paying more for energy as a result of unbalanced voltage supplies. From the power system point of view, operation of motor

Table 4Influence of 3% unbalanced voltage upon the performance of a 7.5 kW three phase induction motor

		0 1	1			1			
Unb. (%)	Ave. (V)	I <sub>a</sub> (Anal.)	I <sub>a</sub> (Exp.)	<i>I</i> <sub>b</sub> (Anal.)	I <sub>b</sub> (Exp.)	<i>I</i> <sub>c</sub> (Anal.)	<i>I</i> <sub>c</sub> (Exp.)	$P_{\rm loss}$ (%)	Eff. (%)
2.97	371	14.71	14.2	18.63	18.2	14.87	14.6	105.16	84.74
3.04	372.7	14.5	14.1	18.57	18.3	14.97	14.7	104.6	84.8
2.99	368	18.02	17.7	16.7	16.3	13.69	13.5	105.66	84.66
3	368	18.31	18	15.95	15.7	14.15	14	105.54	84.67
2.97	370	14.21	14.2	18.31	18	15.62	15.5	104.8	84.78
	Jnb. (%) .97 .04 .99	Jnb. (%)         Ave. (V)           .97         371           .04         372.7           .99         368           .97         370	Jnb. (%)         Ave. (V) $I_a$ (Anal.)           .97         371         14.71           .04         372.7         14.5           .99         368         18.02           .368         18.31           .97         370         14.21	Jnb. (%)         Ave. (V) $I_a$ (Anal.) $I_a$ (Exp.)           .97         371         14.71         14.2           .04         372.7         14.5         14.1           .99         368         18.02         17.7           .68         18.31         18           .97         370         14.21         14.2	Jnb. (%)         Ave. (V) $I_a$ (Anal.) $I_a$ (Exp.) $I_b$ (Anal.)           .97         371         14.71         14.2         18.63           .04         372.7         14.5         14.1         18.57           .99         368         18.02         17.7         16.7           .97         370         14.21         14.2         18.31	Jnb. (%)         Ave. (V) $I_a$ (Anal.) $I_a$ (Exp.) $I_b$ (Anal.) $I_b$ (Exp.)           .97         371         14.71         14.2         18.63         18.2           .04         372.7         14.5         14.1         18.57         18.3           .99         368         18.02         17.7         16.7         16.3           .97         370         14.21         14.2         18.31         18	Jnb. (%)         Ave. (V) $I_a$ (Anal.) $I_a$ (Exp.) $I_b$ (Anal.) $I_b$ (Exp.) $I_c$ (Anal.)           .97         371         14.71         14.2         18.63         18.2         14.87           .04         372.7         14.5         14.1         18.57         18.3         14.97           .99         368         18.02         17.7         16.7         16.3         13.69           .97         370         14.21         14.2         18.31         18         15.62	Jnb. (%)Ave. (V) $I_a$ (Anal.) $I_a$ (Exp.) $I_b$ (Anal.) $I_b$ (Exp.) $I_c$ (Anal.) $I_c$ (Exp.).9737114.7114.218.6318.214.8714.6.04372.714.514.118.5718.314.9714.7.9936818.0217.716.716.313.6913.5.9737014.2114.218.311815.6215.5	Jnb. (%)Ave. (V) $I_a$ (Anal.) $I_a$ (Exp.) $I_b$ (Anal.) $I_b$ (Exp.) $I_c$ (Anal.) $I_c$ (Exp.) $P_{loss}$ (%).9737114.7114.218.6318.214.8714.6105.16.04372.714.514.118.5718.314.9714.7104.6.9936818.0217.716.716.313.6913.5105.66.9618.311815.9515.714.1514105.54.9737014.2114.218.311815.6215.5104.8

with lower efficiency implies an increase of the system load and a reduction of the power plant reserves.

Consideration of the economic loss for the nation is useful because in many countries, electricity must be supplied by the government to the consumers. Therefore, the nation is interested in investing less for establishing new power plants. If the unbalanced voltage could be diminished, less electrical energy is consumed and optimized use of electrical energy is possible. On the other hand, such economic analysis can also be directed toward a large industrial firm that is not run by the government. So, this analysis is beneficial from different points of view.

The load increase of induction motors under voltage unbalance can be expressed as

Load increase rate = 
$$\frac{\text{Input power of motor under voltage unbalance}}{\text{Input power of motor under voltage balance}}$$
 (6)

The load increase rate is a parameter that can be used for an economic analysis of the result of voltage unbalance. To do this study, the number of three phase induction motors operating is needed. The number of motors in Iran between 4 kW and 15 kW that were sold over the period 1988 and 2002 is given in Table 5.

The 7.5 kW motor can be considered as representative of the motors listed in Table 5, i.e. it is assumed that 372,800 three phase induction motors are now operating in the country. Since these motors are 10% of the total motors in the whole country, it is estimated that the total number of motors is  $372,800 \times 10 = 3,728,000$ . An analysis of motors operating under 1–6% voltage unbalance with phase angles of 120° and an average voltage equal to 360 V has been conducted and is summarized in Table 6. The annual electrical energy consumption and increase in costs paid

Statistics of solu motors over period 1700-2002										
	Order									
	1	2	3	4	5					
Output power (W)	4	5.5	7.5	11	15					
Number	139,704	87,864	88,942	35,154	21,160					

Table 5Statistics of sold motors over period 1988–2002

Table 6

Economic losses due to voltage unbalance in the power system

Unbalanced factor (%)	Efficiency (%)	Load increase rate	Extra consumed power (kW)	Extra consumed energy (kW h/year)	Extra payment (Rials/year)
1	84.54	1.0104	340,411	851,027,500	85,102,750,000
2	84.46	1.0110	360,050	900,125,000	90,012,500,000
3	84.32	1.0120	392,782	981,955,000	98,195,500,000
4	84.13	1.0153	500,797	1,251,992,500	125,199,250,000
5	83.89	1.0171	559,714	1,399,285,000	139,928,500,000
6	83.59	1.0212	693,915	1,734,787,500	173,478,750,000

Total installed power 32,731,840 kW, average operation hours 2500.

300

by consumers are given in Table 4. In this study, it is assumed that the average efficiency of the machine under balanced voltage is 85.4%. Based on this assumption, the average power consumption with balanced voltage is 8.78 kW. The cost of 1 kW h electricity is 100 Rials. Table 6 indicates that the additional energy consumed during the year under different voltage unbalances is considerable. It is necessary to know the existing voltage unbalance at any instant in order to specify the total load of the power system. On the other hand, it is necessary to know this voltage unbalance in order to be able to regulate the power generation to cover the extra load due to the voltage unbalances. An increase of the energy costs for a 1% voltage unbalance amounts to  $85 \times 10^9$  Rials.

### 6. Conclusions

The efficiency of three phase induction motors is decreased by increasing the voltage unbalance. This reduction leads to an increase of the total power system losses. The present study shows that if a motor operating under voltage unbalance is properly derated, an increase of the voltage unbalance can even improve the efficiency. In other words, for the same voltage unbalance, a derated motor may have a higher efficiency than a non-derated motor.

Based on the approximate number of existing three phase induction motors in the country, an economical analysis of the results of the voltage unbalance on the performance of the motors and the additional consumed power for different voltage unbalances was conducted. The extra payment by consumers shows that these numbers are very large, which is a strong motivation to find a proper solution for reduction of the losses.

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