Earth Rod Electrode Resistance

by: Ver Pangonilo PEE RPEQ MIEAust

1.1 SINGLE EARTH ROD ELECTRODE

The resistance of a single earth can be calculated if the soil resistivity is known. An earth electrode driven vertically, the electrode resistance can be calculated using the equation

$$R = \frac{\rho}{2\pi L}\left[\ln\left(\frac{8L}{d}\right) - 1\right]$$

where:

R = electrode resistance, Ω

ρ = Soil resistivity, Ω-m (uniform soil[1])

L = Length of electrode buried in soil, m

d = Outer diameter of earth rod, m

*Note: In Australia the minimum depth of vertical type earth electrode is **1.2 m** [Clause 5.3.6.3, AS/NZS 3000 - 2007]*

The soil resistivity at the location of the earth electrode needs to be investigated. The resistivity of soils varies with

1. the depth from the surface,
2. the type and concentration of soluble chemicals in the soil,
3. the moisture content, and
4. the soil temperature.

In other words, the soil resistivity is dependent on the electrolyte in the soil. The presence of surface water does not necessarily indicate low resistivity.

In the absence of soil investigative data, typical resistivity of various soil types is given in the following table.

Table 1 Typical soil resistivity (Source: IEEE 142 / BS 7430)

Soil Type	Average Resistivity *(Ω-m)*
Well-graded gravel	600 – 1000
Poorly graded gravel	1000 – 2500
Clayey gravel	200 – 400
Silty sand	100 – 800
Clayey sands	50 – 200

[1] *Uniform soil means that the soil type is the same for the whole length of the earth electrode*

Soil Type	Average Resistivity *(Ω-m)*
Silty or clayey sand with slight plasticity	30 – 80
Fine sandy soil	80 – 300
Gravelly clays	20 – 60
Inorganic clays of high plasticity	10 – 55
Surface Soils	1 - 50
Clay	2 - 100
Sandy clay	100 - 150
Moist gravel	50 - 700
Dry gravel	700 - 1200
Limestone	5 - 10000
Porous limestone	30 - 100
Quartz, crystalline limestone	100 - 1000
Sandstone	20 - 2000
Granites	900 - 1100
Concrete	300 - 500

Example:

Soil resistivity = 180 Ω-m

Earth rod = 19∅ x 1.8 m

Solution:

Assuming that the earth electrode is fully driven into the uniform soil, the resistance will be

$$R = \frac{\rho}{2\pi L}\left[\ln\left(\frac{8L}{d}\right) - 1\right]$$

$$R = \frac{180}{2\pi(1.8)}\left[\ln\left(\frac{8(1.8)}{(19/1000)}\right) - 1\right]$$

$$R = 89.6\Omega$$

1.2 MULTIPLE EARTH ROD ELECTRODES

In the event that the desired soil resistivity is not achievable using a single earth electrode, multiple electrodes could be used.

The combined resistance of rod electrodes in parallel can be obtained from the following equation:

$$R_n = R\left(\frac{1+\lambda a}{n}\right)$$

In which

$$a = \frac{\rho}{2\pi Rs}$$

Where

R = single rod resistance in isolation, Ω

S = rod spacing, m

ρ = Soil resistivity, Ω-m

λ = multiplying factor (see Table 2 or Table 3)

n = number of electrodes (see Table 2 or Table 3)

The above equations assume that rod electrodes can be represented approximately by hemispherical electrodes, having the same earth resistance. This assumption is satisfactory provided that the spacing between the rods is less than their length.

Table 2 Factors for parallel electrodes in line (BS 7430)

Number of electrodes (*n*)	Factor (λ)
2	1.00
3	1.66
4	2.15
5	2.54
6	2.87
7	3.15
8	3.39
9	3.61
10	3.81

For electrodes equally spaced around a hollow square, e.g. around the perimeter of a building, the equations given above are used with a value of λ taken from Table 3.

For three rods placed in an equilateral triangle, or in an L formation, a value of λ = 1.66 may be assumed.

Table 3 Factors for electrodes in a hollow square (BS 7430)

Number of electrodes (*n*)	**Factor (λ)**
2	2.71
3	4.51
4	5.48
5	6.13
6	6.63
7	7.03
8	7.36
9	7.65
10	7.90
12	8.32
14	8.67
16	8.96
18	9.22
20	9.44

Note: The total number of electrodes around the square is 4(n-1).

The reduction in combined earth resistance provided by additional electrodes inside the square is small, but such electrodes will reduce the potential gradient over the soil surface inside the square. A practical example of this is the use of strip electrodes forming an earth grid within the square.

Table 3 may also be used for electrodes arranged in a rectangle, where n is given by (total number of electrodes/4) + 1. Provided that the length to width ratio of the rectangle does not exceed 2, the error will be less than – 6 %.

EXAMPLE

From the previous example, let us calculate the earth resistance if we use 10 rod electrodes in line arrangement.

R = 89.6 Ω

ρ = 180 Ω-m

L = 1.8 m

The spacing should be less than the rod length ($s < L$). Assume ***s = 1.5 m***.

$$a = \frac{\rho}{2\pi R s} = \frac{180}{2\pi(89.6)(1.5)} = 0.2132$$

$$R_n = R\left(\frac{1 + \lambda a}{n}\right)$$

Assume 10 electrodes. From Table 2, λ=3.81.

$$R_n = 89.6\left(\frac{1+3.81\times 0.2132}{10}\right) = 16.23\Omega$$

If we change the arrangement to a hollow square arrangement, 10 electrodes in Table 3, λ=7.90.

$$R_n = 89.6\left(\frac{1+7.90\times 0.2132}{10}\right) = 24.05\Omega$$

1.3 APPARENT SOIL RESISTIVITY FOR TWO-LAYER SOIL

In locations where the soil type is not homogeneous, the resulting resistivity for a two-layer soil can be calculated using the formula

$$\rho(a) = \rho_1\left[1+4\sum_{n=1}^{\infty}\frac{K^n}{\sqrt{1+\left(2n\frac{h}{a}\right)^2}} - \frac{K^n}{\sqrt{4+\left(2n\frac{h}{a}\right)^2}}\right]$$

where:

$$K = \frac{\rho_2-\rho_1}{\rho_2+\rho_1}$$

ρ_1 : Top layer soil resistivity

ρ_2 : Deep layer soil resistivity

h : Depth of top layer soil

h : Depth of top layer soil

a : Earth electrode spacing

n : Number of earth electrodes

1.4 FORMULAE FOR CALCULATION OF EARTH RESISTANCES

	Hemisphere radius a	$R = \frac{\rho}{2\pi a}$
•	One ground rod length L, radius a	$R = \frac{\rho}{2\pi L}\left(\ln\frac{4L}{a} - 1\right)$
• •	Two ground rods $s > L$; spacing s	$R = \frac{\rho}{4\pi L}\left(\ln\frac{4L}{a} - 1\right) + \frac{\rho}{4\pi s}\left(1 - \frac{L^2}{3s^2} + \frac{2L^4}{5s^4}\cdots\right)$
••	Two ground rods $s < L$; spacing s	$R = \frac{\rho}{4\pi L}\left(\ln\frac{4L}{a} + \ln\frac{4L}{s} - 2 + \frac{s}{2L} - \frac{s^2}{16L^2} + \frac{s^4}{512L^4}\cdots\right)$
—	Buried horizontal wire length $2L$, depth $s/2$	$R = \frac{\rho}{4\pi L}\left(\ln\frac{4L}{a} + \ln\frac{4L}{s} - 2 + \frac{s}{2L} - \frac{s^2}{16L^2} + \frac{s^4}{512L^4}\cdots\right)$
└	Right-angle turn of wire length of arm L, depth $s/2$	$R = \frac{\rho}{4\pi L}\left(\ln\frac{2L}{a} + \ln\frac{2L}{s} - 0.2373 + 0.2146\frac{s}{L} + 0.1035\frac{s^2}{L^2} - 0.0424\frac{s^4}{L^4}\cdots\right)$
⅄	Three-point star length of arm L, depth $s/2$	$R = \frac{\rho}{6\pi L}\left(\ln\frac{2L}{a} + \ln\frac{2L}{s} + 1.071 - 0.209\frac{s}{L} + 0.238\frac{s^2}{L^2} - 0.054\frac{s^4}{L^4}\cdots\right)$
+	Four-point star length of arm L, depth $s/2$	$R = \frac{\rho}{8\pi L}\left(\ln\frac{2L}{a} + \ln\frac{2L}{s} + 2.912 - 1.071\frac{s}{L} + 0.645\frac{s^2}{L^2} - 0.145\frac{s^4}{L^4}\cdots\right)$
✳	Six-point star length of arm L, depth $s/2$	$R = \frac{\rho}{12\pi L}\left(\ln\frac{2L}{a} + \ln\frac{2L}{s} + 6.851 - 3.128\frac{s}{L} + 1.758\frac{s^2}{L^2} - 0.490\frac{s^4}{L^4}\cdots\right)$
✳	Eight-point star length of arm L, depth $s/2$	$R = \frac{\rho}{16\pi L}\left(\ln\frac{2L}{a} + \ln\frac{2L}{s} + 10.98 - 5.51\frac{s}{L} + 3.26\frac{s^2}{L^2} - 1.17\frac{s^4}{L^4}\cdots\right)$
○	Ring of wire diameter of ring D, diameter of wire d, depth $s/2$	$R = \frac{\rho}{2\pi^2 D}\left(\ln\frac{8D}{d} + \ln\frac{4D}{s}\right)$
▬	Buried horizontal strip length $2L$, section a by b, depth $s/2$, $b < a/8$	$R = \frac{\rho}{4\pi L}\left(\ln\frac{4L}{a} + \frac{a^2 - \pi ab}{2(a+b)^2} + \ln\frac{4L}{s} - 1 + \frac{s}{2L} - \frac{s^2}{16L^2} + \frac{s^4}{512L^4}\cdots\right)$
	Buried horizontal round plate radius a, depth $s/2$	$R = \frac{\rho}{8a} + \frac{\rho}{4\pi s}\left(1 - \frac{7}{12}\frac{a^2}{s^2} + \frac{33}{40}\frac{a^4}{s^4}\cdots\right)$
	Buried vertical round plate radius a, depth $s/2$	$R = \frac{\rho}{8a} + \frac{\rho}{4\pi s}\left(1 + \frac{7}{24}\frac{a^2}{s^2} + \frac{99}{320}\frac{a^4}{s^4}\cdots\right)$

Source : Table 4-5 IEEE 142 - 2007

References

1. IEEE 142 (Green Book) 2007 - IEEE Recommended Practice for Grounding of Industrial and Commercial Power Systems
2. IEEE 81 - 1983 - IEEE Guide for Measuring Earth Resistivity, Ground Impedance, and Earth Surface Potentials of a Ground System
3. BS 7430 - 1998 Code of practice for Earthing
4. Practical Grounding, Bonding, Shielding and Surge Protection
   G. Vijayaraghavan, Mark Brown, Malcolm Barnes
   Newnes 2004 – IDC Technologies
5. AS/NZS 3000 – 2007 Wiring Rules